Parallel Electric Field of a Mirror Kinetic Alfvén Wave

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Abstract
It has been shown that an Alfvén wave can create an electric field along the magnetic field line due to mirror effect. In a converging magnetic field, mobility of electrons is restricted by the mirror force and a parallel electric field must exist to carry a field aligned current associated with Alfvén waves. The parallel electric field created by this mechanism can be much larger than that of a kinetic or inertial Alfvén wave in the low frequency limit.
1. Introduction

In basic MHD theory it is assumed that an electric field along a magnetic field line (let us call this "parallel electric field"); in this paper words parallel/perpendicular are used with respect to the ambient magnetic field) is negligibly small because of the high mobility of electrons in the parallel direction. Magnetosphere-ionosphere coupling models based on MHD theory assume that the magnetospheric potential is directly mapped onto the ionosphere by field-aligned currents (parallel currents) carried by Alfvén waves. The parallel electric field is out of scope of these MHD models. There are some models that include the parallel electric field due to anomalous resistivity, however, this parallel field is assumed to be created by high-frequency turbulent waves, not by MHD waves.

In contrast, there have been theories that predicts the existence of parallel electric fields due to mirror force in magnetosphere-ionosphere coupling process. In a converging magnetic field, electrons' downward motion is restricted by the mirror force and requires a finite parallel electric field for a parallel current. Using a one-dimensional static model, Knight [1973] has derived a relation that the parallel potential drop is approximately proportional to the parallel current.

The result of Knight [1973] clearly contradicts with the MHD models. According to Kinght's theory, on the one hand, there must be a parallel electric field for the existence of a field-aligned current in the static limit. On the other hand, it is possible to consider the static limit of the MHD models, and a field-aligned current requires no parallel electric field there. This inconsistency comes from the fact that MHD theory cannot treat the mirror force properly. An Alfvén wave must have a parallel electric field in a mirror field to be consistent with the Kinght's theory.

The parallel electric field generation by Alfvén waves has been investigated in a different context. Strictly speaking there exists a parallel electric field even in ordinary MHD waves, but its amplitude is negligibly small for waves with MHD scales because of the high mobility of electrons in the parallel direction. However, when the perpendicular scale of an Alfvén wave becomes smaller than the MHD scale, the parallel electric field becomes no more negligible. This kind of Alfvén wave is called "kinetic Alfvén wave." As seen from the above argument the wave must be extremely localized in the perpendicular direction to have finite parallel electric field. Also the parallel electric field of this wave exists only with a time varying field aligned current. Therefore, this wave is applicable to small scale, transient phenomena only.

There are two types of so called "kinetic Alfvén wave" depending on the electron thermal speed. When the electron thermal speed is much higher than the Alfvén speed, what causes the parallel electric field is the electron pressure, [Hasegawa, 1976], and in the opposite case (electron thermal speed is much lower), electron inertia creates the parallel electric field [Goeltz and Boswel, 1979]; each type of kinetic Alfvén waves has different dispersion relation. Historically both types are called "kinetic Alfvén wave," however, this name is not appropriate for the latter type because the kinetic effect is not important for this wave. Therefore, we use the name "inertial Alfvén wave" found in recent papers [Thompson and Lysak, 1996; Náse et al. 1998] for this type of waves.

The calculation of the kinetic or inertial Alfvén wave has been done assuming a homogeneous ambient magnetic field in the past literature, therefore the mirror effect has not been considered so far. In the present paper, we examine the parallel electric field generation of Alfvén waves by the mirror effect. We call this type of waves "mirror kinetic Alfvén wave."

Our results show that the mirror kinetic Alfvén wave can create parallel electric field more effectively than the kinetic or inertial Alfvén waves. Unlike kinetic or inertial Alfvén waves, the parallel electric field exists even after the transient time of the Alfvén wave.

The structure of this paper is the following. We firstly review the mechanism of the kinetic and inertial Alfvén waves in Section 2. This is helpful for understanding the parallel electric field generation by the mirror kinetic Alfvén wave, which is presented in Section 3. To treat the wave propagation in a mirror field self-consistently, one inevitably needs to include inhomogeneity of the background magnetic field. The computation required for the self-consistent treatment would be extremely difficult, and perhaps a numerical experiment is the only pragmatic way. In this paper we employ test wave approach that is not self-consistent, but can illustrate the effect of mirror force qualitatively.

Auroral particle acceleration is the one of the most important applications of the parallel electric field in magnetospheric physics. In Section 4, we briefly examine the applicability of the mirror kinetic Alfvén wave to this problem by comparing it with the other models of parallel electric field generation. Concluding remarks will be given in Section 5.
2. Kinetic and Inertial Alfvén Waves

In this section we briefly review the mechanism of the parallel electric field generation by the kinetic or inertial Alfvén wave in a homogeneous background. The analysis here is essentially the same as the one by Nakamura [1989] and can be regarded as a special case of more general solutions obtained by Lysak [1996].

Suppose a plasma in a uniform ambient magnetic field; we take the $z$ axis of Cartesian coordinates along the ambient field. The $x$ axis is in the direction of the perpendicular electric field, and $\partial/\partial y = 0$ is assumed since we are interested in the shear Alfvén mode. In the following, symbols have conventional meaning ($j$: current; $t$: time, etc.) unless otherwise stated.

Neglecting the displacement current we have $\nabla \times \mathbf{E} = \partial j/\partial t$ from the Maxwell’s equations:

$$k_x^2 E_x - k_z k_x E_z = \mu_0 \omega j_x$$

(1)

In this section physical quantities ($E_z$, $j_x$ etc.) should be understood as Fourier (space) - Laplace (time) transformed values as in the standard procedure in wave analysis. In MHD time scale ($\omega \ll \text{ion cyclotron frequency}$), the perpendicular current $j_x$ is carried by the polarization current, thus we can write

$$j_x = \frac{\omega}{\mu_0 V_A^2} E_x$$

(2)

where $V_A$ is the Alfvén speed. This polarization current is carried by the polarization drift of ions; electrons are strongly magnetized and their motion in the $x$ direction is negligible. Substituting Eq. (2) into Eq. (1) yields

$$\left( k_x^2 - \frac{\omega^2}{V_A^2} \right) E_x + k_z k_x E_z = 0$$

(3)

When $k_x$ is so small that the second term is negligible, the above equation gives the dispersion relation of ordinary Alfvén waves. When the second term is not negligible, we need one more equation to relate $E_x$ to $E_z$.

The parallel current $j_z$ is predominantly carried by electrons because of their high mobility in the parallel direction. The parallel dynamics of electrons can be expressed by the linearized guiding center Vlasov equation [e. g. Fejer and Kan, 1969] as

$$\left( \frac{\partial}{\partial t} + v_z \frac{\partial}{\partial z} \right) \delta f = \frac{eE_z}{m_e} \frac{\partial}{\partial v_z} f_0$$

(4)

where $\delta f$ and $f_0$ are the first and zeroth order distribution function of electrons, and $e$ and $m_e$ are the charge and mass of a electron. Electrons’ guiding center has a perpendicular velocity component in the $y$ direction due to the $E \times B$ drift, however, it does not contribute to the charge neutral condition because $\partial/\partial y = 0$. Also the current due to the electron $E \times B$ motion is canceled by the same drift of ions. Therefore we neglect the perpendicular motion in the above expression.

The parallel current $j_z$ can be calculated form Eq. (4) as $j_z = e n_0 \int \delta v_z f dv_z$ and the perpendicular current $j_x$ is given by Eq. (2). We can obtain the ratio of $E_z$ to $E_x$ by combining these two equations with the quasi-neutral condition $k_x j_x + k_z j_z = 0$. For a Maxwellian distribution of electrons ($f_0 = (\pi^{-1/2}/v_{th}) \exp(-v_z^2/v_{th}^2)$) the ratio is

$$\frac{E_z}{E_x} = \frac{1}{1 - \frac{\omega}{k_z k_x} Z \left( \frac{\omega}{k_z v_{th}} \right) V_A^2 \omega_{pe}^2}$$

(5)

where $\omega_{pe}$: plasma frequency, $c$: speed of light, and $Z$ is the plasma dispersion function defined as $Z(s) = (1/\sqrt{\pi}) \int_{-\infty}^{\infty} \exp(-\zeta^2)/(s-\zeta)d\zeta$ (see, e. g., Stix [1992], Baumjohan and Treumann [1996]). This expression has the following meaning. The Alfvén wave propagation causes a time varying perpendicular electric field $E_x$ as in Eq. (3) and the polarization current $j_x$ is invoked by $E_x$. When $k_z \neq 0$ the divergence of the polarization current $k_x j_x$ results in the charge excess, which must be neutralized by the parallel motion of electrons to satisfy the quasi-neutral condition. A parallel electric field $E_z$ must exist to cause the electron parallel motion here, but it is negligibly small when $k_x$ is small enough, which is the case for the classical MHD theory.

Let us examine under what condition $E_z$ is be negligible in Eq. (3). We consider two limiting cases: low temperature ($V_A \gg v_{th}$) and high temperature ($V_A \ll v_{th}$) limits. Each limit corresponds to the inertia and kinetic Alfvén wave respectively. The wave frequency $\omega$ can be estimated by $V_A k_z$ and we can make use of the Taylor (low temperature) and asymptotic (high temperature) expansion for the $Z$ function in each limit. Then we obtain

$$\frac{E_z}{E_x} \approx \frac{\omega^2 c^2 k_z^2}{V_A^2 \omega_{pe}^2} \kappa_z \left( \frac{c k_z}{\omega_{pe}} \right)^2 \frac{k_z}{k_x} \quad (V_A \gg v_{th})$$

$$\frac{E_z}{E_x} \approx \left( \frac{c k_z v_{th}}{V_A \omega_{pe}} \right)^2 \frac{k_z}{k_x} \quad (V_A \ll v_{th})$$

(6)
When the electron thermal speed is much lower than the Alfvén speed, \(E_z\) must accelerate electrons to neutralize the charge excess by \(k_x j_x\), thus the parallel field \(E_z\) is balanced by the inertia of electrons. In this case \(E_z\) becomes negligible when \(k_x \ll c/\omega_{pe}\).

In the opposite limit, i.e., the thermal speed is much higher than the Alfvén speed, the pressure becomes dominant in the electron dynamics; what sustains \(E_z\) is the electron pressure in this case. The condition to neglect \(E_z\) is \(k_x \ll c v_t / \omega_{pe} \Lambda_A\). When the ion temperature is equal to the electron pressure, this condition can be rewritten \(k_x \ll \rho_i\), where \(\rho_i\) being the thermal Larmor radius of ions. This expression is widely used in the past literature because of its handiness, however, the ion dynamics has nothing to do with this as seen from the argument above. As long as the electrons have finite temperature, the parallel field \(E_z\) can exist even when the ion temperature is zero.

In both (high and low temperature) cases, the parallel electric field is negligibly small when the Alfvén wave has an MHD spatial scale. This is why the parallel electric fields are treated as zero in MHD theory.

3. Mirror kinetic Alfvén Wave

In this section we include the mirror force and examine its effect on the parallel electric field generation. The propagation of an Alfvén wave is essentially driven by the \(E_\perp\) combined with \(j_\perp\) by the Maxwell's equations as shown in Eq. (3). The parallel field \(E_\parallel\) couples the wave propagation with the electron dynamics resulting a slight modification of the wave. Therefore, we treat the \(E_\parallel\) propagating at the Alfvén speed (i.e., \(E_\parallel\) calculated without the electron dynamics) as the zero-th order approximation and estimate \(E_\parallel\) resulting from it.

We concentrate on the situation where the electron thermal speed is lower than the Alfvén speed because this is the case important for the auroral particle acceleration.

3.1. Mechanism for Parallel Electric Field

To begin with, we briefly outline how an inertial Alfvén wave form a closed current loop to satisfy quasi-neutrality in a homogeneous magnetic field. To illustrate this, let us suppose a constant voltage generator has been turned on at one end of the magnetic field as illustrated in Figure 1. The perpendicular electric field \(E_\perp\) due to this voltage generator propagates as an Alfvén wave along the magnetic field. At

the wave front a polarization current \(j_\perp\) is invoked by \(\partial E_\perp/\partial t\). As seen in Section 2, \(j_\perp\) causes charge excess when \(k_x \neq 0\), and it causes \(E_\parallel\); electrons are accelerated by this \(E_\parallel\) to keep quasi-neutrality.

It should be noted that \(j_\perp\) and the resulting \(E_\parallel\) exists only where the temporal variation of \(E_\perp\) exists. Therefore, the acceleration of electrons has been done only once at the wave front, and after that electrons move at the constant speed with their inertia behind the wave front. This inertial motion of electrons creates \(j_\parallel\), which is connected to \(j_\perp\) at the wave front. This is how a closed current loop is formed to keep quasi-neutrality.

Now let us consider the case with the mirror effect. In the presence of the mirror force, an electron accelerated downward by \(E_\parallel\) will be mirrored back after the mirroring time with the same accelerated speed in the opposite (upward) direction. Thus the current carried by the electrons will be considerably reduced when the wave frequency is lower than the characteristic time scale of the mirroring. Therefore a parallel electric field must exist behind the wave front to carry the field aligned current required to compensate the polarization current at the wave front.

The lower the wave frequency becomes, the larger the parallel electric field must be. This can be understood by considering the limit where the wave frequency is zero. When the wave frequency is zero, i.e., the static limit, the electron downward flux accelerated by the parallel electric field is the same as the mirrored upward flux, if there were no loss cone effect; the parallel electric field must be infinitely large to cause a finite current in this case. In the actual magnetosphere the parallel electric field can be finite because of the loss cone effect as shown by Knight [1973], but it must be much larger for lower frequency waves. We do not take the loss cone effect into account in the following calculation. It should be considered in the next step of this study.

3.2. Calculation

Suppose a converging magnetic field line illustrated in Figure 2. A reservoir that supplies an isotropic Maxwellian plasma to the system is located at one end; this corresponds to the plasma sheet in the magnetosphere-ionosphere coupling problem. We assume a local model that represents a localized region on this magnetic field line; the size of this region is much smaller than the overall magnetic field length. With this local model we neglect the effects of inhomogeneity other than the mirror force. This is not
that the other effects are negligible compared to the mirror force. Our aim is to investigate the effect of mirror force qualitatively and the other effects are expected not to alter the result essentially even though they are not negligible. We leave more self-consistent calculation including other inhomogeneity effects to future studies. With the same reason, we neglect the calculation including other inhomogeneity effects to allow the region of our interest is localized and far enough from the reservoir, and thus the above expression is valid for a sufficiently long time scale compared to the wave propagation. Substituting the above expression into Eq. (8) yields

\[
\delta f = e^{ik_z(z-V_A t)} \frac{eE_z}{m_e} \frac{\partial f_0}{\partial v_z} \int_0^t dt \exp[-ik_z(V_A - v_z)\tau - ik_zv_z^2\tau^2]
\]

where \( \epsilon = \frac{1}{2} \partial \ln B/\partial z \). The integration in the above expression can be carried out analytically with a function \( G(s, \eta, \alpha) \) defined as following:

\[
G(s, \eta, \alpha) = \int_0^s d\zeta \exp[i(1 + i0 - \eta)\zeta + i\alpha\zeta^2]
\]

\[
= \frac{(-1)^{\frac{1}{4}} \sqrt{\pi}}{2\sqrt{\alpha}} \exp\left[-\frac{i(\eta - 1)^2}{4\alpha}\right] \frac{\epsilon}{k_z V_A^2} \frac{v}{V_A} \sum_{\eta, \alpha} \left\{ \operatorname{erf}\left[\frac{(-1)^{\frac{1}{4}}(1 - \eta)}{2\sqrt{\alpha}}\right] - \operatorname{erf}\left[\frac{(-1)^{\frac{1}{4}}(1 - \eta + 2\alpha s)}{2\sqrt{\alpha}}\right] \right\}
\]

where \( \operatorname{erf} \) is the error function defined as \( \operatorname{erf}(s) = \int_0^s \exp(-s^2) d\zeta \). The term \( i0 \) in the integrand corresponds to the imaginary part of the wave frequency that comes from the Landau contour. With this term the above integration converges at \( s \to \infty \). We can write Eq. (11) as

\[
\delta f = e^{ik_z(z-V_A t)} \frac{eE_z}{m_e} \frac{\partial f_0}{\partial v_z} \int_0^t dt \exp[ik_z(\xi(z, v; t - t') - V_A t')]
\]

where \( \xi(z, v; \tau) \) represents the particle trajectory; a particle, whose position is \( z \) at a time \( t \), was at \( \xi \) time \( \tau \) ago.

Neglecting the variation of the mirror force along the field line in our local model, \( \xi \) can be written as

\[
\xi(z, v; \tau) = z - v_z\tau - \frac{\mu}{2m} \frac{\partial B}{\partial z} \tau^2
\]

where \( \mu = m v_z^2/2B_z \). This expression becomes invalid when \( \tau \) is so large that \( \xi \) reaches the reservoir in Figure 2. We assume that the region of our interest is localized and far enough from the reservoir, and thus the above expression is valid for a sufficiently long time scale compared to the wave propagation.
Physically speaking, the time dependence in $G$ in Eq. (12) represents the initial disturbance due to the particle individual motion [Balescu, 1963]; this disturbance will decay out after the initial transient time and we are not interested in it. What we are interested in is the part due to the wave propagation, which is represented by the factor $\exp(-V_A t)$ in Eq. (12). Therefore we replace $G$ with $G_\infty$ in the following calculation. This justifies the assumption that both $E_x$ and $E_z$ have the same form of propagation as in Eq. (7).

With a Maxwellian background distribution, $f_0 = (1/\sqrt{\pi} v_{th}^3) \exp[-(v_x^2 + v_z^2)/v_{th}^2]$, the charge excess due to the electrons can be expressed as

$$
\delta n_e = 2\pi e n_0 \int_0^\infty dv_L v_L \int_0^\infty dv_z \delta f = \frac{2e^2 n_0 E_z}{\sqrt{\pi} m_e v_{th} k_z V_A} \int_0^\infty dv_L v_L \int_0^\infty dv_z \delta f \tag{14}
$$

$$
v_z G_\infty \left(\frac{v_z}{V_A}, \frac{v_z^2}{k_z V_A^2}\right) \exp\left(-\frac{v_x^2 + v_z^2}{v_{th}^2}\right)
$$

The integration over $v_z$ in the above equation can be carried out with a method developed by Nakamura and Hoshino [1998], which makes use of the following rational approximation to the Gauss function.

$$
\exp(-c^2) \approx C_0 \prod_{k=1}^N (\zeta - c_k)(\zeta + c_k) \tag{15}
$$

where asterisk (*) indicates complex conjugate and we choose the notation of $c_k$ as $\text{Im}(c_k) \geq 0$. The result is

$$
\int_{-\infty}^{\infty} dv_z v_z G_\infty \left(\frac{v_z}{V_A}, \frac{v_z^2}{k_z V_A^2}\right) \exp\left(-\frac{v_x^2 + v_z^2}{v_{th}^2}\right)
$$

$$
= \sum_{j} \frac{\pi C_0 v_{th}^2}{\prod_{k\neq j}(c_j^2 - c_k^2)} G_\infty \left(\frac{c_j v_{th}}{V_A}, \frac{c_j v_{th}^2}{k_z V_A^2}\right) \tag{16}
$$

Combining Eq. (15) and Eq. (2) with the quasi-neutral condition and substituting the above expression, we obtain the ratio of $E_z$ to $E_x$ as:

$$
\frac{E_z}{E_x} = \left[ \frac{\alpha_p^2 v_A^2}{c^2 k_x v_{th}^2} \int_0^\infty d\nu \nu e^{-\nu^2} \right] \int_0^\infty d\nu \nu e^{-\nu^2}
$$

$$
\sum_{j} \frac{C_0}{\prod_{k\neq j}(c_j^2 - c_k^2)} G_\infty \left(\frac{c_j v_{th}}{V_A}, \frac{c_j v_{th}^2}{k_z V_A^2}\right)^{-1}
$$

We should be careful to carry out the above integration numerically because numerical convergence of $G_\infty(\eta, \alpha)$ becomes poor when $\alpha \ll 1$; in the following calculation we use a Taylor expansion form for $\alpha \ll 1$.

3.3. Result

Figure 3 is the ratio $|E_z/E_x|$ plotted against the parallel wave number $k_z$ with several different electron thermal speeds. The ratio is normalized as to be unity when the mirror effect is absent. We see that the parallel electric field is enhanced due to the mirror effect when $k_z$ is small, or when the wave frequency is low, equivalently.

Also we see from Figure 3 that the parallel electric field becomes larger for larger thermal speed. This is because that the larger perpendicular speed means the larger mirror force and the larger parallel speed means the shorter mirroring time. This point would be important for the location of the parallel electric field in the magnetosphere. The electron temperature is higher in the high altitude magnetosphere, however, the mirror ratio is small. The mirror force becomes large near the ionosphere, but the temperature is low. Both situations may not be suitable for the parallel electric field; the parallel electric field would be generated in the region in-between.

To demonstrate the parallel electric field generation mechanism we give an example of a current pattern in the following form:

$$
E_x = E_{x0}[\text{erf}(z - V_A t) - \text{erf}(z - V_A t - L)] \tag{18}
$$

which is schematically illustrated in Figure 4a. This represents propagating currents loop with length $L$ in the parallel direction; the structure is periodic in the perpendicular direction since we pick up one Fourier component in $x$. The reason why we choose the above shape is that this is the simplest example that can self-consistently illustrate the parallel electric field generation mechanism.

In Figure 4b $E_z$ patterns for $\epsilon \neq 0$ (with mirror effect) and $\epsilon = 0$ (without mirror effect) are plotted with an illustrated current loop with $L = 5.0/\epsilon$. A remarkable difference is that a finite parallel electric field $E_x$ exists throughout the current loop with the presence of the mirror effect. A longer current loop can create larger potential drop.
4. Comparison with Other Mechanisms

In this section we examine the applicability of the mirror kinetic Alfvén wave to the auroral acceleration problem. The aim of this paper is to point out the existence of mirror kinetic Alfvén wave, and we have revealed only its very basic mechanism. Thus it is impossible to draw some definite conclusion from our analysis; before that we need further theoretical analysis of the mechanism, as well as much more careful comparison with the observational results. What we outline below is just implications of our result so far.

There have been proposed a number of mechanisms for the auroral electron acceleration; for example, twelve different mechanisms has been listed in the review by Borovsky [1993]. Here in this section we discuss the four major ones among them, namely weak double layers, anomalous resistivity, inertial Alfvén waves, and static mirror effects.

4.1. Weak Double Layers

Satellite observations found the existence of small electric spikes on auroral field lines, which is considered as “weak double layers” [e. g., Koskinen and Mälkiä, 1993; Eriksson and Boström, 1993, and references therein]. Early simulations on double layers [Hubbard and Joyce, 1979] had assumed a fixed potential drop between the simulation boundaries, and found that the applied potential drop tend to localize into double layers as time goes on. Later, this type of simulations were regarded to be inappropriate for auroral physics and current-driven type of simulations took place of them. This is because what is needed to be explained is, or seemed to be, the generation of the potential drop itself.

However, with mirror kinetic Alfvén waves, the simulations with fixed potential drop can explain the observed double layer-like structures. A mirror kinetic Alfvén wave creates a large scale potential drop as we have seen in this paper, and that can be approximated by the fixed potential drop of simulations. Then, as shown by the simulations, electrostatic effects will create localized double layers, which are observed by the satellites. Double layers do not have to create potential drop but can be a result of a large scale potential drop, and then the parallel electric field of mirror kinetic Alfvén waves is consistent with the observation.

4.2. Inertial Alfvén Waves

The parallel electric field generation by inertial Alfvén waves have been extensively investigated in connection with the auroral particle acceleration [e. g., Lysak, 1998 and references therein]. Especially inertial Alfvén waves are considered to be important because the electron thermal seed at the auroral acceleration region is lower than the Alfvén speed.

An significant difference of the parallel electric field created by a mirror kinetic Alfvén wave is in the point that the electric field can be time stationary. The parallel field by an inertial Alfvén wave is sustained by the electron inertia, and thus it can exist only when ∂E⊥/∂t ≠ 0 as can be seen in Figure 4. Therefore the particle acceleration by inertial Alfvén wave is inevitably time-dependent and the acceleration by a wave packet continues no longer than the wave transit time through the acceleration region.

Observations indicate that the acceleration region is localized in the spatial scale of the order of several thousand kilometers along the field line, thus the duration of the particle acceleration is at most a little more than ten seconds; such short-lived acceleration cannot explain at least observed quiet auroral arcs. Even for moving active arcs, it is difficult to explain their motion by the bouncing of inertial Alfvén waves. In contrast a mirror kinetic Alfvén wave generates the parallel field even where ∂E⊥/∂t = 0, and thus can accelerate particles continuously.

Another advantage of the mirror kinetic Alfvén wave is the amplitude of the parallel field. The perpendicular wave length of a inertial Alfvén wave must be extremely small, comparable to the electron skin depth namely, to accelerate electrons effectively. A mirror kinetic Alfvén wave can create much larger parallel electric field when the parallel wave number is small (frequency is low) as illustrated in Figure 3.

4.3. Anomalous Resistivity

The consequences of parallel electric field created by mirror kinetic Alfvén waves may have similar feature to the one created by anomalous resistivity. What restricts the parallel mobility of electrons are high frequency turbulent waves in the anomalous resistivity model, instead of the mirror force. The anomalous resistivity can also create time stationary parallel electric field, and thus can continuously accelerate electrons.

Magnetosphere-ionosphere coupling models based on shear Alfvén waves with anomalous resistivity (e.
the relation by Knight wave should be coincide with the linearized version of current-potential relation of a mirror kinetic Alfvén wave. Actually, in the time stationary limit, the same as the one in static models with the mirror field by a mirror kinetic Alfvén wave is essentially a turbulence but more like a series of weak double layers.

4.4. Static Mirror Effect

The basic mechanism to generate parallel electric field by a mirror kinetic Alfvén wave is essentially the same as the one in static models with the mirror effect. Actually, in the time stationary limit, the current-potential relation of a mirror kinetic Alfvén wave should be coincide with the linearized version of the relation by Knight [1973], if the loss cone effect is included. In a sense the theory in this paper can be regarded as a generalization of the theory with mirror effect for time dependent and multi-dimensional cases in connection with Alfvén waves.

This generalization has a great advantage. By its nature a static model cannot explain the relation of cause and effect; one cannot tell how the potential drop has been formed as a result of the magnetosphere-ionosphere interaction. We can investigate dynamic process of parallel electric field generation when we know the time dependent aspect of the parallel electric field generation mechanism.

5. Concluding Remarks

It has been shown that an Alfvén wave can have a finite parallel electric field because of the mirror effect. Unlike kinetic or inertial Alfvén waves, the parallel electric field exist in the static limit. This mechanism can be a new candidate for the potential drop along auroral field lines.

The scope of the present paper is to qualitatively show the mechanism of parallel electric field generation, and the calculation is over-simplified for the quantitative evaluation with the realistic parameters. Here we only give a rough estimation on the condition under which the mirror effect becomes important for an Alfvén wave.

The term $k_\perp \varepsilon v^2 \tau^2$ represents the mirror effect in Eq. (11), therefore the mirror effect becomes prominent when $v_\perp \varepsilon \sim \varepsilon v^2 \tau^2$. We can estimate $\tau$ by the wave period $1/\omega = 1/V_\parallel k_\perp$, and $v_\perp$ and $v_\parallel$ can be estimated by the electron thermal speed $v_{th}$. The above condition becomes $\omega \sim \varepsilon v_{th}$. With the order estimation of $v_{th} \sim 100km/sec$ and $1/\varepsilon \sim 1000km$ for the auroral acceleration region, we see that the mirror effect is prominent for the time scale longer than ten seconds.

Alfvén waves are basic mechanism of the magnetosphere-ionosphere coupling, and mirror force inevitably plays a role in this coupling process because of the converging magnetic field above the ionosphere. Therefore the generation mechanism of the parallel electric field investigated in this paper can be important in phenomena other than auroral electron acceleration.

For example, Nóse et al., [1998] have investigated the electron precipitation associated with geomagnetic ULF waves; they compared the observed features with those predicted by three theories: (1) drift-bounce resonance, (2) ULF-VLF-particle interaction, and (3) kinetic Alfvén waves. Their conclusion is that the observed acceleration of precipitating electrons is best explained by kinetic Alfvén waves. However, the energy of observed precipitation electrons is closer to the upper limit of the one estimated with the inertial Alfvén waves, in other words, a mechanism that can create a larger electric field can explain the observation better. Mirror effect can create an electric field much larger than the inertia effect as we have seen in this paper, and thus our theory can explain the result of Nóse et al., [1998] better.

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**Figure 1.** Schematic illustration of the parallel electric field generation by Alfvén waves. In the presence of the mirror effect, parallel electric field must exist behind the wave front to sustain the field aligned current.

**Figure 2.** The ratio $|E_z/E_x|$ plotted against the parallel wave number $k_z$ with several different electron thermal speeds. The thermal speeds are scaled by the Alfvén speed $V_A$.

**Figure 3.** a) Schematic illustration of the current loop; b) Shape of the parallel electric field created by the current loop.
a) Current Pattern

![Current Pattern Diagram]

b) E parallel (without mirror)

c) E parallel (with mirror)
\[ \left| \frac{E_z}{E_x} \right| \]

\[ \log_{10} \left( \frac{k_z}{\varepsilon} \right) \]
Figure 4

(a) Diagram showing a cycle with arrows indicating direction.

(b) Graph plotting $E_z$ (arbitrary unit) against $z$ (scaled in $\varepsilon^{-1}$).